

题号

4.5 P80

1 曲线 $y = \frac{x+3\sin x}{4x-5\cos x}$ 的水平渐近线

解: $\lim_{x \rightarrow \infty} \frac{x+3\sin x}{4x-5\cos x} = \frac{1}{4}$, $y = \frac{1}{4}$ 为水平渐近线.

2 曲线 $y = \frac{x^2-3x+2}{x^2-1}$ 的垂直渐近线为

解: $x^2-1=0 \Rightarrow x=-1$ 或 $x=1$

$$\lim_{x \rightarrow -1} \frac{x^2-3x+2}{x^2-1} = \lim_{x \rightarrow -1} \frac{(x-1)(x-2)}{(x-1)(x+1)} = \infty \quad \lim_{x \rightarrow 1} \frac{x^2-3x+2}{x^2-1} = \frac{-1}{2}$$

于是 $x=-1$ 为曲线 $y = \frac{x^2-3x+2}{x^2-1}$ 的垂直渐近线.

3 曲线 $xy=4$ 在点 $(2,2)$ 处的曲率 $k =$

解: $xy=4 \Rightarrow y = \frac{4}{x}$, $y' = -\frac{4}{x^2}$, $y'' = \frac{8}{x^3}$, $y'(2) = -1$, $y''(2) = 1$

$$k = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{1}{[1+(-1)^2]^{3/2}} = \frac{\sqrt{2}}{4}$$

4 曲线 $y = \sin x$ 在点 $(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$ 处的曲率半径为 $\rho =$

解: $y' = \cos x$, $y'' = -\sin x$, $y'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$, $y''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$$k = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{\frac{\sqrt{2}}{2}}{[1+(\frac{\sqrt{2}}{2})^2]^{3/2}} = \frac{\sqrt{2}}{3}, \quad \rho = \frac{1}{k} = \frac{3\sqrt{2}}{2}$$

1 求曲线 $y = x \ln(e + \frac{1}{x})$ 的斜渐近线.

解: $a = \lim_{x \rightarrow \infty} \frac{y}{x} = \lim_{x \rightarrow \infty} \frac{x \ln(e + \frac{1}{x})}{x} = \lim_{x \rightarrow \infty} \ln(e + \frac{1}{x}) = 1$

$$b = \lim_{x \rightarrow \infty} (y - ax) = \lim_{x \rightarrow \infty} [x \ln(e + \frac{1}{x}) - x]$$

$$= \lim_{x \rightarrow \infty} x [\ln(e + \frac{1}{x}) - \ln e] = \lim_{x \rightarrow \infty} x \ln(1 + \frac{1}{ex})$$

$$= \lim_{x \rightarrow \infty} x \cdot \frac{1}{ex} = \frac{1}{e}$$

所以 $y = ax + b = x + \frac{1}{e}$ 为曲线的斜渐近线.

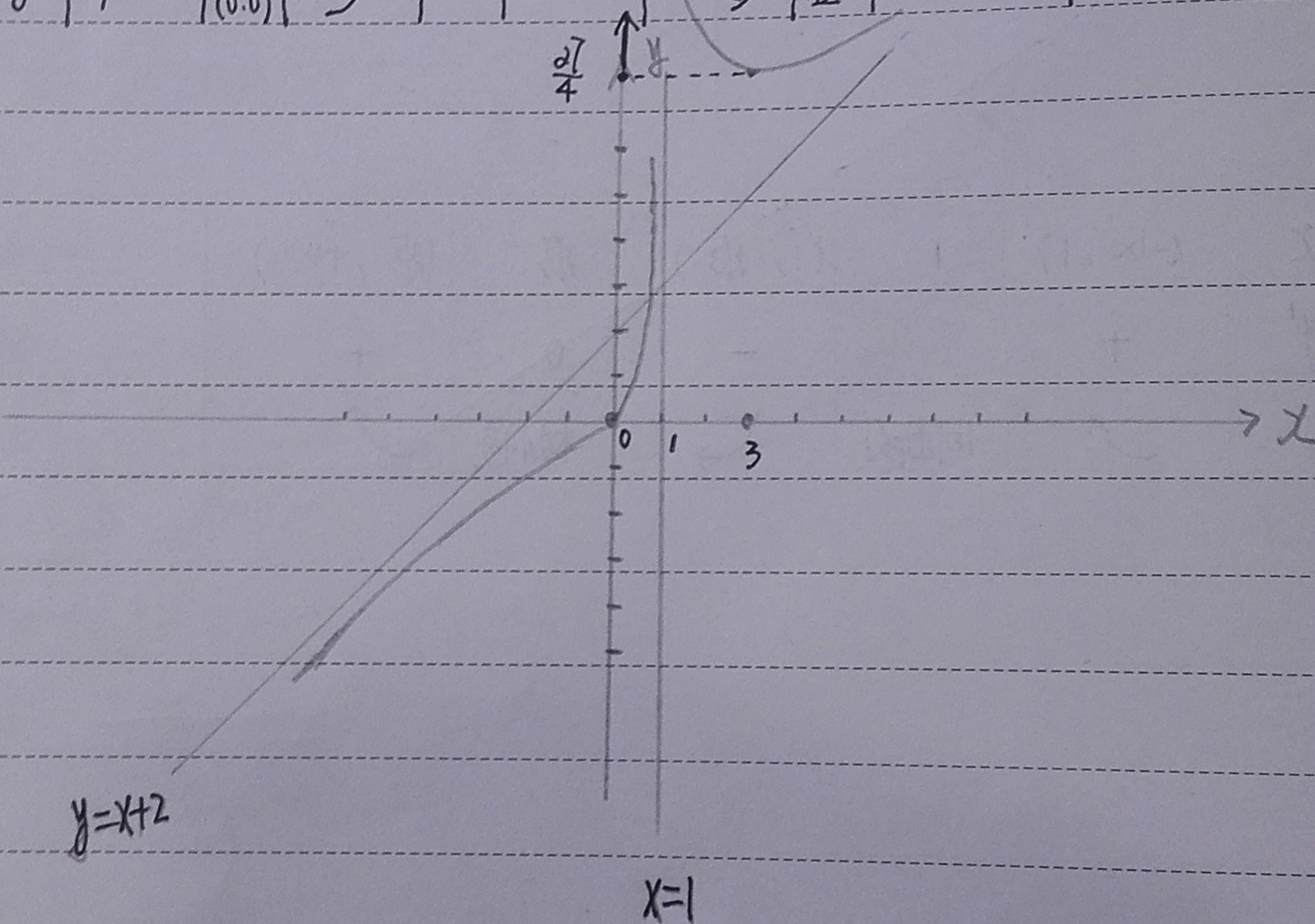
二 2

描绘 $y = \frac{x^3}{(x-1)^2}$ 的图形解: $y = \frac{x^3}{(x-1)^2}$ 的定义域为 $(-\infty, 1) \cup (1, +\infty)$ $\lim_{x \rightarrow 1} \frac{x^3}{(x-1)^2} = +\infty$, $x=1$ 为 $y = \frac{x^3}{(x-1)^2}$ 的垂直渐近线 $\lim_{x \rightarrow \infty} \frac{y}{x} = \lim_{x \rightarrow \infty} \frac{x^3}{x(x-1)^2} = 1$, $\lim_{x \rightarrow \infty} (y-x) = \lim_{x \rightarrow \infty} \frac{x^3 - (x-1)^2 x}{(x-1)^2} = 2$, $y = x+2$ 为 $\frac{x^3}{(x-1)^2}$ 的斜渐近线

$$y' = \frac{3x^2(x-1)^2 - 2(x-1) \cdot x^3}{(x-1)^4} = \frac{x^3 - 3x^2}{(x-1)^3} \quad y'' = \frac{(3x^2 - 6x)(x-1)^3 - 3(x-1)^2(x^3 - 3x^2)}{(x-1)^6} = \frac{6x}{(x-1)^4}$$

令 $y' = 0 \Rightarrow x=0$ 或 $x=3$, 令 $y'' = 0 \Rightarrow x=0$ 或 $x=\frac{1}{3}$, 列表

x	$(-\infty, 0)$	0	$(0, \frac{1}{3})$	$\frac{1}{3}$	$(\frac{1}{3}, 1)$	$(1, 3)$	3	$(3, +\infty)$
y'	+	0	+	—	—	—	0	+
y''	—	0	+	—	—	+	+	+
y	↗	拐点 (0,0)	↗	—	—	↘	极小 值	↗



题号	一	二	三	四	五	六	七	八	九	十	总分
得分											
阅卷人											

考生注意: 答题时应注明题号, 并按题号顺序答题。

题号	答题内容
P81 4.5节	廖国勇
二 3	求 $\begin{cases} x = \ln(1+t^2) \\ y = t - \arctan t \end{cases}$ 在对应 $t=1$ 点处的曲率 解: $dx/dt = \frac{2t}{1+t^2}$ $\frac{d^2x}{dt^2} = \frac{2(1+t^2) - (2t)^2}{(1+t^2)^2}$ $x'(1) = \frac{2}{2} = 1$ $x''(1) = \frac{4-4}{4} = 0$ $y'(t) = 1 - \frac{1}{1+t^2}$ $y''(t) = \frac{2t}{(1+t^2)^2}$ $y'(1) = \frac{1}{2}$ $y''(1) = \frac{1}{2}$ $K = \frac{ x'(t)y''(t) - x''(t)y'(t) }{\{[x'(t)]^2 + [y'(t)]^2\}^{3/2}} \Big _{t=1} = \frac{\frac{1}{2}}{(1+\frac{1}{4})^{3/2}} = \frac{4\sqrt{5}}{25}$ (注, 参考答疑为 $\frac{\sqrt{6}}{9}$, 是错误的)
4	曲线 $y = x^2 - 2x$ 上哪点的曲率最大? 并求最大曲率. 解: $y' = 2x - 2$ $y'' = 2$ $K(x) = \frac{ y'' }{(1+y'^2)^{3/2}} = \frac{2}{[1+(2x-2)^2]^{3/2}}$ $= \frac{2}{(4x^2-8x+5)^{3/2}}$ $K'(x) = 2(-\frac{3}{2}) \cdot (4x^2-8x+5)^{-\frac{5}{2}} (8x-8)$ 令 $K'(x) = 0 \Rightarrow x = 1$, $x < 1$ 时 $K'(x) > 0$, $x > 1$ 时 $K'(x) < 0$ 所以 $x=1$ 为 $K(x)$ 的极大值点, 且是唯一的极大值点, 该点就是最大值点, 此时 $K(1) = \frac{2}{(4-8+5)^{3/2}} = 2$, 综上知 曲线在 $(1, -1)$ 点曲率最大, 最大曲率为 2. 求曲线 $x^2 + xy + y^2 = 3$ 在点 $(1, 1)$ 处的曲率及曲率半径.
5	解: 方程两边对 x 求导 $2x + y + xy' + 2yy' = 0$ (1) 把 $x=1, y=1$ 代入得 $y'(1) = -1$, 在 (1) 式两边对 x 求导 $2 + y' + y' + xy'' + 2y'y' + 2yy'' = 0$ 把 $x=1, y=1, y'=-1$ 代入得 $y''(1) = -\frac{2}{3}$ 于是 $K _{(1,1)} = \frac{ y''(1) }{[1+y'(1)^2]^{3/2}} = \frac{\frac{2}{3}}{2\sqrt{2}}$ $= \frac{\sqrt{2}}{6}$, $\rho = \frac{1}{K} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$